Math 10A
Worksheet, Discussion #25; Friday, 7/20/2018
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1 Recurrence Relations

1.1 Concepts

1. A **homogeneous** recurrence relation does not include any extra constants (e.g. $a_n = a_{n-1} + a_{n-2}$) and a **nonhomogeneous** recurrence relation contains one (e.g. $a_n = a_{n-1} + 4$). The **order** of a recurrence relation is the "farthest" back the relation goes. For instance, the order of $a_n = a_{n-1} + a_{n-3}$ is 3 because we need the term 3 terms back (a_{n-3}). A **linear** recurrence relation has all the a_i terms being linear and a recurrence relation with **constant coefficients** is one where the coefficients in front of the a_i are all constants.

1.2 Problems

2. For the following recurrence relations, find their order and label them as homogeneous, linear, and/or with constant coefficients.

   (a) $a_n = a_{n-1} + na_{n-1}^2$

   (b) $a_n = n^2 a_{n-1} - a_{n-2}$

   (c) $a_n = 4a_{n-1} - 2a_{n-4} + 3$

   (d) $a_n = a_{n-1}^2 - n^2$

   (e) $a_n = a_{n-2}$

   (f) $a_n = a_{n-1} - a_{n-2}$

Solution:

	(a)	(b)	(c)	(d)	(e)	(f)
order	1	2	4	1	2	2
homogeneous?	yes	yes	no	no	yes	yes
linear?	no	yes	yes	no	yes	yes
constant coefficients?	no	no	yes	yes	yes	yes

3. Find constants A, B such that $a_n = An + B$ is a solution to the recurrence relation $a_n = 2a_{n-1} - 3a_{n-2} + 2n$.

Solution: We plug in $a_n = An + B$ to get

$$An + B = 2(A(n-1) + B) - 3(A(n-2) + B) + 2n =$$

$$2An - 2A + 2B - 3An + 6A - 3B + 2n = (-A+2)n + 4A - B.$$

So, we get that $A = -A+2$ or $A = 1$ and $B = 4A-B$ or $B = 2A = 2$. So $a_n = n+2$.

4. Verify that $a_n = n + 1$ is a solution to $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$.

Solution: We plug in $a_n = n + 1$ to get

$$n + 1 = 3(n - 1 + 1) - 3(n - 2 + 1) + (n - 3 + 1) = n + 3 - 2 = n + 1.$$

So it is a solution to the recurrence relation.

5. Find constants A, B such that $a_n = An + B$ is a solution to the recurrence relation $a_n = a_{n-1} + a_{n-3} + n + 3$.

Solution: We plug in $a_n = An + B$ to get

$$An + B = A(n-1) + B + A(n-3) + B + n + 3 = (2A+1)n - 4A + 2B + 3.$$

So, we get that $A = 2A+1$ or $A = -1$ and $B = -4A+2B+3$ or $B = 4A-3 = -7$. So $a_n = -n - 7$.

6. Verify that $a_n = 1 - n$ is a solution to $a_n = 2a_{n-1} - a_{n-2}$.

Solution: We plug in $a_n = 1 - n$ to get

$$1 - n = 2(1 - (n-1)) - (1 - (n-2)) = 2 - 2n + 2 - 1 + n - 2 = -n + 1.$$

So it is a solution to the recurrence relation.